Math 22 Quiz 7 Version Wed Nov 21, 2012 (DUE Mon Nov 26, 2012 @ START OF CLASS)

SCORE: ____ / 35 POINTS

- Sign below to confirm that the work shown on this quiz is strictly your own work.
- You may have consulted your textbook, your notes and the class handouts, and used your calculator,
- but you did NOT consult other people, websites, software or other outside sources of help.

SIGNATURE:

You ONLY need to write formal proofs when specifically instructed to

In lecture, we defined $2Z = \{2x \mid x \in Z\}$. Let $kZ = \{kx \mid x \in Z\}$ for each positive integer k. SCORE: ____/ 8 PTS

Prove that, for all $m, n \in Z^+$, mZ and nZ have the same cardinality by finding a one-to-one correspondence $f: mZ \to nZ$.

You must WRITE A FORMAL PROOF that your function *f* is one-to-one and onto. You do NOT need to prove *f* is a function. HINT: Start by trying to find a one-to-one correspondence from 2*Z* to 3*Z*, then generalize it.

Let *R* be a relation on *Z* defined by xRy if and only if $(x - y) \mod 4 = 2$.

SCORE: / 8 PTS

[a] Is *R* transitive ? If yes, write a formal proof. If no, give a counterexample and show clearly how it indicates *R* is not transitive.

[b] Is *R* symmetric? If yes, write a formal proof. If no, give a counterexample and show clearly how it indicates *R* is not symmetric.

Let R be the equivalence relation on $A = \{1, 2, 3, 5, 6, 8, 12\}$ defined by

xRy if and only if $\frac{x}{y} = 2^n$ for some $n \in Z$

Find the partition induced on A by R. You do not need to prove that R is an equivalence relation.

Let R be an equivalence relation on set A. Write a formal proof for the following statement. SCORE: ____/ 6 PTS Use the definitions in sections 8.2 and 8.3 but do NOT use any of the lemmas, theorems or homework exercises as justification.

For all $a, b, c \in A$, if $[a] \cap [b] \neq \emptyset$ and $c \in [b]$, then $c \in [a]$.

Let $A = Z^+ - \{1\}$. Define $f : A \to Z^{nonneg}$ by the rule

 $f(x) = 71 \mod x$

[a] Find f(9).

[b] Is f one-to-one? Explain briefly.

[c] Is f onto ? Explain briefly.

[d] Find $f^{-1}(\{1\})$. Write your answer in set roster notation. HINT: Write your answer in set builder notation first and use the definition of *mod.* SCORE: ____ / 9 PTS